Mar. 22, 2017

Sect. 6-6a

Identity Matrix

Inverse of a Matrix

(Square)

Given A, find A^{-1}

Identity Matrix
(Square Matrix)

$$I = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix}$$

$n \times n$

$$A \cdot I = A$$

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \underline{2} & \underline{-1} \\ \underline{3} & \underline{4} \end{bmatrix}$$

$$I \cdot A = A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \underline{2} & \underline{-1} \\ \underline{3} & \underline{4} \end{bmatrix}$$

Inverse Matrix

Notation:

Given matrix A

A^{-1} "A inverse"

Property of Inverse

$A \cdot A^{-1} = \underline{I}$

$A^{-1} \cdot A = \underline{I}$

Are $A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$ inverses? (Y or N)

$$AB = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \underline{1} & \underline{0} \\ \underline{0} & \underline{1} \end{bmatrix}$$

Yes, A & B are inverses.

$$BA = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \underline{1} & \underline{0} \\ \underline{0} & \underline{1} \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \quad \text{find } A^{-1}$$

$$\left[\begin{array}{cc|cc} -1 & 2 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] -R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|cc} 1 & -2 & -1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] R_1 + R_2 \rightarrow R_2 \qquad \left[\begin{array}{cc|cc} 1 & -2 & -1 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right] -R_2 \rightarrow R_2$$

$$\left[\begin{array}{cc|cc} 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & -1 \end{array}\right] \quad 2R_2 + R_1 \to R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

So

$$A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \quad \text{find } B^{-1}$$

$$\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow{-R_1+R_2 \to R_2} \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right] \xrightarrow{2R_2+R_1 \to R_1}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right] \quad B^{-1} = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -1R_1 + R_2 \rightarrow R_2 \\ -6R_1 + R_3 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 + R_1 \rightarrow R_1 \\ \\ -4R_2 + R_3 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right]$$

$R_3 + R_1 \rightarrow R_1$

$R_3 + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right]$$

So

$$A^{-1} = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix} \quad \text{find } C^{-1}$$

$$\left[\begin{array}{ccc|ccc} -2 & -3 & 1 & 1 & 0 & 0 \\ -3 & -3 & 1 & 0 & 1 & 0 \\ -2 & -4 & 1 & 0 & 0 & 1 \end{array}\right] \quad -R_2 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ -3 & -3 & 1 & 0 & 1 & 0 \\ -2 & -4 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} 3R_1 + R_2 \to R_2 \\ 2R_1 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -3 & 1 & 3 & -2 & 0 \\ 0 & -4 & 1 & 2 & -2 & 1 \end{array}\right] -R_3 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -4 & 1 & 2 & -2 & 1 \end{array}\right] 4R_2 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 6 & -2 & -3 \end{array}\right]$$

$$C^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

Inverse of 2 × 2 Shortcut

$$\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \Rightarrow \frac{1}{\det}\begin{bmatrix} -5 & 3 \\ -2 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix}^{-6}_{-5} = -5-(-6) = 1$$

$$\begin{bmatrix} 5 & -3 \\ 1 & -2 \end{bmatrix} \begin{array}{l} -3 \\ \\ \\ -10 \end{array} \Rightarrow \frac{1}{-7}\begin{bmatrix} -2 & 3 \\ -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{7} & -\frac{3}{7} \\ \frac{1}{7} & -\frac{5}{7} \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 \\ 1 & 2 \end{bmatrix} \Rightarrow \frac{1}{-4}\begin{bmatrix} 2 & 0 \\ -1 & -2 \end{bmatrix}$$